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MINIMIZING LINEAR COUPLING IN THE AGS WITH SKEW QUADS

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Introduction

Skew quadrupoles and solenoids can introduce linear betatron coupling into the beam. Linear coupling causes an energy exchange between the horizontal and vertical betatron motions. Such a process can lead to vertical emittance growth during the adiabatic betatron tune crossing. When two linear oscillators are coupled, one can find normal modes. The motion of these two normal modes are uncoupled. There exists separate Courant- Snyder invariants for these two normal modes. When the motion is projected onto $x - z$ planes, there is a smear. The betatron tunes of these two normal modes are separated by

$$\Delta\nu = \sqrt{(\nu_x - \nu_z)^2 + \mid C_- \mid^2}$$

where

$$C_- = \frac{1}{2\pi} \int \sqrt{\beta_x \beta_z} e^{i[\mu_x - \mu_z - (\nu_x - \nu_z - p)\theta]} [\frac{a_1}{\rho} + \frac{M_s}{2}(\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{\beta_z}) - i\frac{M_s}{2}(\frac{1}{\beta_x} + \frac{1}{\beta_z})] ds$$

where

$$a_1/\rho = \frac{\partial B_z}{\partial z}/B\rho, \qquad M_s = B_s/B\rho,$$

are skew quadrupole and solenoid strength, respectively, p is an integer nearest to $\nu_x - \nu_z$.

The linear coupling causes emittance growth, so it should be minimized. Linear coupling caused by a solenoid can be corrected with skew quadrupoles by minimizing the coupling coefficient C_-. It requires two orthogonal skew quadrupole families.

The major source of linear coupling arises from the 5% partial solenoidal snake to be installed in the section I10 of the AGS. To correct the effect of linear coupling, we will use 4 families of skew quadrupoles located at (1) B17, D17, ...,L17; (2) A-L6; (3) A-L16; (4)A-L3. The A6 and A16 families are called weak skew quadrupoles.

C_- of the Partial Snake

For a snake, the coupling constant is

$$C_- = \frac{M_s}{4\pi} \int \sqrt{\beta_x \beta_z} e^{i[\mu_x - \mu_z - (\nu_x - \nu_z - p)\theta]} [(\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{\beta_z}) - i(\frac{1}{\beta_x} + \frac{1}{\beta_z})]\theta_{sol}(s)ds$$

$$= \frac{M_s}{4\pi} \int_{s_0}^{s_0+C} \sqrt{\beta_x \beta_z} e^{i[\mu_x - \mu_z - (\nu_x - \nu_z - p)\frac{s}{R}]} [(\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{\beta_z}) - i(\frac{1}{\beta_x} + \frac{1}{\beta_z})]\theta_{sol}(s)ds$$

where the solenoid strength for the 5% snake is $M_s = 0.05 \times 1.125/2.4384 = 0.023m^{-1}$, with $2.4384m$ as the length of the solenoid, s is the distance from the beginning point, and $\theta_{sol}(s)$ is a function defined as

$$\theta_{sol}(s) = \begin{cases} 1 & \text{if s within snake} \\ 0 & \text{otherwise.} \end{cases}$$

$\nu_x = 8.75, \nu_z = 8.75, R = \frac{C}{2\pi} = \frac{807.11}{2\pi} = 128.46m$. Using the Simpson rule, the integral from the snake is

$$C_- \mid_{sn} = -0.01216 - i0.00844 = 0.0148(-0.8216 - i0.5703)$$

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For $\nu_x = 8.66667, \nu_z = 8.66667$, the coupling coefficient is

$$C_- \mid_{sn} = -0.01075 - i0.0101 = 0.01475(-0.7288 - i0.6824)$$

It shows that the coupling coefficient $C_- \mid_{sn}$ is not sensitive to the betatron tune changes.

C_- of the Skew Quadrupoles

For skew quadrupoles, since a set of skew quadrupoles use a common power supply, they have the same strength. Using the thin lens approximation, the coupling constant is given by

$$
\begin{aligned}
C_- &= [\frac{a_1}{\rho}]\frac{1}{2\pi} \int \sqrt{\beta_x \beta_z} e^{i[\mu_x - \mu_z - (\nu_x - \nu_z - p)\theta]} ds \\
&\doteq [\frac{a_1}{\rho}\Delta s]\frac{1}{2\pi} \sum_{i=1}^{N} \sqrt{\beta_x(s_i)\beta_z(s_i)} e^{i[\mu_x(s_i) - \mu_z(s_i) - (\nu_x - \nu_z - p)\frac{s_i}{R}]}
\end{aligned}
$$

where p is the integer part of $\nu_x - \nu_z$. N=6 for skew quadrupoles in section 17, N=12 for skew quadrupoles in section 3, 6 and 16. Then we get the C_- for every skew quadrupoles family:

$$C_- \mid_{sk3} = a_3(28.6566 - i2.9010) = a_3 28.8031(.9949 - i.1007),$$

$$C_- \mid_{sk6} = a_6(29.0680 - i6.1395) = a_6 29.7093(.9704 - i.2067),$$

$$C_- \mid_{sk16} = a_{16}(29.9289 + i0.6021) = a_{16} 29.935(.9998 + i.0201),$$

$$C_- \mid_{sk17} = a_{17}(14.5170 - i1.1866) = a_{17} 14.5654(.9967 - i.0814)$$

where $a_i = [\frac{a_1}{\rho}\Delta s]$ is the strength for every set of skew quadrupoles.

Minimizing the Total C_-

Minimizing the coupling requires at least two skew quadrupole families. First we consider the two 'strong' skew quadrupoles. It can be seen from the

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above equations that $C_- \mid_{sk3}$ and $C_- \mid_{sk17}$ are almost in phase. Since there are 20 sections in each superperiod, the skew quadrupoles in sections 3 and 17 are in symmetric positions, which gives rise to the in-phase effect. Due to this in-phase effect, we cannot just use the two 'strong' quadrupoles in sections 3 and 17 to compensate the coupling. From now on, we just consider the skew quadrupoles in sections 6, 16 and 17.

Cancel the coupling means $\sum C_- = 0$. Writing the real and imaginary parts individually, we get two linear equations:

$$14.5170a_{17} + 29.9289a_{16} + 29.0680a_6 - 0.01216 = 0 \tag{1}$$

$$-1.1866a_{17} + 0.6021a_{16} - 6.1395a_6 - 0.00844 = 0 \tag{2}$$

The solutions of these linear equations are not unique. To use the skew quadrupoles most efficiently, the minimum values of skew quadrupoles strengths should be used. It is a linear-programming problem. Since there are only three parameters, we can use a simpler way to solve it.

Rewrite the first equation as

$$a_{17} = \frac{1}{14.5170}(0.01216 - 29.9289a_{16} - 29.0680a_6)m^{-1},$$

which is a plane in 3-D space as shown in Fig.1. Then substitute it into the second equation, we get

$$a_6 = 0.2657(-0.00744 + 3.0485a_{16})m^{-1},$$

which gives a straight line as shown in Fig.2. Minimum strengths corresponds to the point with smallest distance from the original point. It is $a_{16} = $
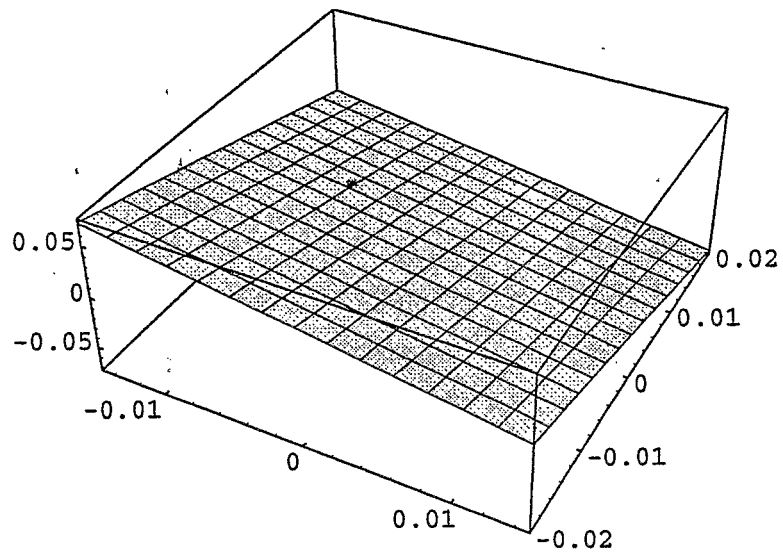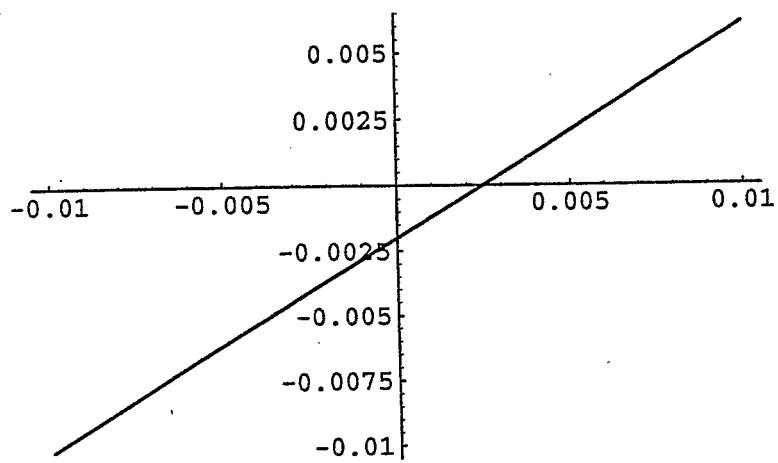
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Figure 1:

Figure 2:

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$0.00097m^{-1}$, $a_6 = -0.0012m^{-1}$. Then a_{17} can be calculated from the first equation: $a_{17} = 0.00124m^{-1}$.

Unfortunately, skew quadrupoles in sections 6 and 16 are 'weak' ones, the strength required cannot be achieved. The intrinsic resonance with highest energy corresponds to $G\gamma = 36 + \nu_z = 44.75$, i.e., $p = 23.4GeV/c$ and $B\rho = 78.0T - m$. The maximum current of the skew quadrupoles in section 17 is 500A, corresponding $\int \frac{\partial B_z}{\partial z} ds = 0.84T$. So the maximum value of a_{17} can not exceed

$$\frac{0.84}{78.0} = 0.0108m^{-1}.$$

The value required for a_{17} is smaller than this. The B fields of the skew quadrupoles in sections 6 and 16 are $0.0084T$, so the maximum value of a_{16} is

$$\frac{0.0084}{78.0} = 0.00011m^{-1}$$

Obviously, this is much smaller than the values required to cancel the coupling coefficient C_- of the snake. It means that the coupling can not be canceled by using the existing skew quadrupoles.

Discussion

To compensate the coupling caused by the 5% snake, new strong skew quadrupole families need to be installed. We need at least two families. To compensate the coupling efficiently, the skew quadrupole families should be located at positions with as large a phase difference as possible. Note that the coupling coefficient $C_- |_{sol}$ from the solenoid has about equal magnitudes of real and imaginary parts, while the coupling coefficients from the quadrupole families are dominantly real except in the 2nd, 6th, 14th and 18th straight

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sections. Since the coupling coefficients from these four straight sections are almost in-phase, we can choose only one of them, say the 6th. Another one should have as large a phase difference as possible from the 6th straight section. Actually, families 6 and 16 have the largest phase difference. So, the 'weak' skew quadrupoles in families 6 and 16 should be replaced by 'strong' skew quadrupoles. The strengthes can be calculated easily by using eqs.(1) and (2):

$$a_{16} = 0.00122m^{-1}, \qquad a_6 = -0.00159m^{-1} \qquad (3)$$

This means that the total skew quadrupole strength needed at the 6th and the 16th straight sections are

$$12a_{16} = 0.0146m^{-1}, \qquad 12 \mid a_6 \mid = 0.0191m^{-1}, \qquad (4)$$

i.e., we need only two strong skew quadrupoles to be located at the 6th and at the 16th straight sections to correct the linear coupling.

In addition to the coupling caused by the 5% snake, there also exists coupling caused by quadrupole misalignment. But the latter one is real part dominant and easier to be compensated for by the skew quadrupole families. The real coupling caused by both of them will be measured during the test run in August, 1993. After that, we will be able to determine the necessary coupling compensation scheme.